One's Complement

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Given an $n-$bit binary string, I, the leftmost bit indicates the sign of an integer in 1s complement representation. In this left most position a 1 indicates a negative value while a 0 indicates a positive value. The representation for positive integers corresponds to unsigned representation where the leftmost bit must contain a 0.

Negative integers are formed by reversing all bits to form the bitwise complement of the corresponding positive integer. If we represent I by the $n-$bit binary sequence, $b_n \ldots b_1$ then $-I$ in one's complement is given by $\overline{b_n} \ldots \overline{b_1}$ where $\overline{b_i} = 1 - b_i$ for all i.

Let's see what that looks like in Math speak

Let I be a negative one's complement integer. The value of I is obtained by forming its one's complement:

$$-I = \sum_{i=0}^{n-1} (1 - b_i) \cdot 2^i = \sum_{i=0}^{n-1} 2^i - \sum_{i=0}^{n-1} b_i \cdot 2^i. \tag{1}$$

Thus,

$$I = \sum_{i=0}^{n-1} b_i \cdot 2^i - (2^n - 1). \tag{2}$$

Negative one's complement integers are formed by subtracting a bias of $2^n - 1$ from the positive integers. Taking into account the sign bit bn, the value for a positive or negative (n+1) bit one's complement integer is:

$$I = \sum_{i=0}^{n-1} b_i \cdot 2^i - b_n(2^n - 1). \tag{3}$$

Recalling that the left most bit only represents the sign, the range of values for an $n-$bit one's complement integer is $-(2^{n-1} - 1)$ to $2^{n-1} - 1$.

Examples:

Since the complement of 0 is $2^{n+1} - 1$, there are different representations for $+0$ and -0 in one's complement. Examples of 8-bit one's complement numbers:

Binary	*Decimal*
00000000	0
11111111	-0
00000011	3
11111100	-3

The range of $8-$bit one's complement integers is -127 to $+127$.

Addition of signed numbers in one's complement is performed using binary addition with end-around carry. If there is a carry out of the most significant bit of the sum, this bit must be added to the least significant bit of the sum.

To add decimal 17 to decimal -8 in 8-bit one's complement:

$$
\begin{array}{rrrrcr}
 & & 0001 & 0001 & & (17) \\
+ & & 1111 & 0111 & & (-8) \\
\hline
 & 1 & 0000 & 1000 & & \\
 & & \hookrightarrow & +1 & & \\
\cline{3-4}
 & & 0000 & 1001 & = & (9)
\end{array}
$$